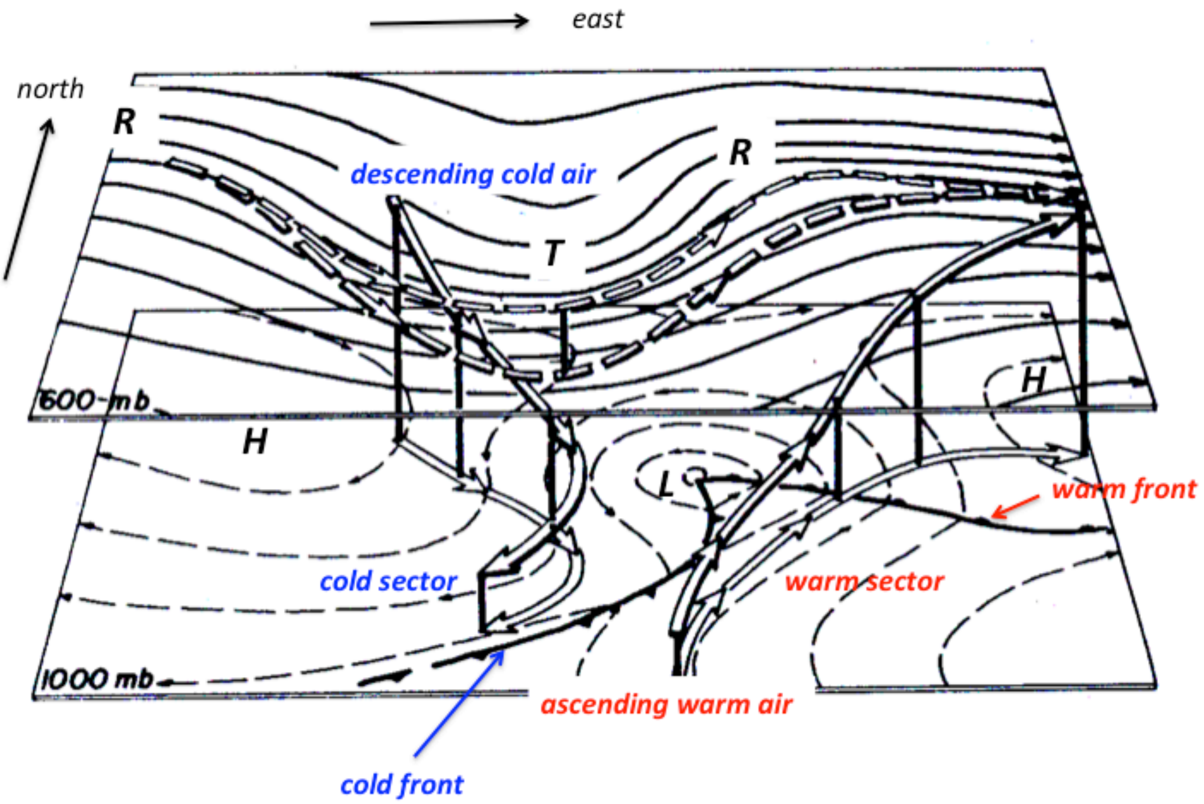
ATMOSPHERIC DYNAMICS

AARNOUT VAN DELDEN

INSTITUTE FOR MARINE AND ATMOSPHERIC RESEARCH
UTRECHT UNIVERSITY
PRINCETONPLEIN 5
3584 CC UTRECHT, THE NETHERLANDS

Lago di Bolsena (Italy): view of growing puffy (cumulus) clouds over the mountains to the west of the lake on 25 July 2010, 17:23 LT.

Lago di Bolsena (Italy): view of a thunderstorm over the mountains to the west of the lake on 25 July 2010, 18:24 LT. The rain shaft is visible below the darkest clouds and a shield of *high* Cirrus clouds (the anvil of the thunderstorm) is visible aloft.



Typical flow pattern between two isobaric surfaces (1000 hPa is near the Earth's surface; 500 hPa is at about 5 km above sea level) in a wave-like disturbance of the jet-stream in the northern hemisphere. The warm air rises along very slanted trajectories, giving rise to cloud layers with a large horizontal scale, as shown in the figure below. See **FIGURE 1.43**.

Cloud layers (stratus) at the background in the "*The Scream*" (1893) by Edvard Munch (1863-1944).

Chapter titles of these lecture notes

Introductory (this document)

In depth (online)

Lecture notes for the course on *Dynamical Meteorology* (NS-MO402M), *Climate Dynamics* (NS-363B) *Boundary Layers, Transport and Mixing* (NS-MO412M) and *Simulation of Ocean, Atmosphere and Climate* (NS-MO501M) at Utrecht University in the academic year 2019-2020.

By Aarnout van Delden

Office: Room 615, Buys Ballot Gebouw (BBG), Princetonplein 5, 3584CC Utrecht, Netherlands
E-mail: a.j.vandelden@uu.nl
Website: http://www.staff.science.uu.nl/~delde102/AtmosphericDynamics.htm

0
Preface

<u>On understanding a complicated problem</u>
Anyone who wants to analyze the properties of matter in a real problem might want to start by writing down the fundamental equations and then try to solve them mathematically. Although there are people who try to use such an approach, these people are the failures in this field; the real successes come to those who start from a *physical* point of view, people who have a rough idea where they are going a then begin by making the right kind of approximations, knowing what is big and what is small in a given complicated situation. These problems are so complicated that even an elementary understanding, although inaccurate and incomplete, is worth while having, and so the subject will be one that we shall go over again and again, each time with more accuracy...
Richard Feynman (1963) in *The Feynman Lectures on Physics,* chapter 39. vol. 1.

i Introduction to atmospheric dynamics

These lecture notes have grown out of many years of giving the one-semester courses on *Geophysical Fluid Dynamics*, *Dynamical Meteorology* and *Climate Dynamics* at Utrecht University. I have given the lecture notes the more natural title *Atmospheric Dynamics*, which covers two subjects that are traditionally called *Dynamical Meteorology* and *Synoptic Meteorology*, as well as part of the subject of *Climate Dynamics*.

Climate Dynamics deals with the theory of the annual average or seasonal average and/or zonal average[1] state of the atmosphere, and usually also involves the ocean, the cryosphere (the glaciers and ice caps) and the biosphere.

Dynamical Meteorology deals with the theory of circulations associated with atmospheric weather phenomena such as cyclones, anticyclones, fronts, sea breezes, tornadoes, thunderstorms, lee-waves and severe downslope windstorms.

Synoptic Meteorology deals with the analysis of observations of these weather phenomena. This analysis is practically impossible without "**conceptual models**" of weather phenomena, or "synoptic models" as they are called on page 231 of Godske et al. (1957)[2]. The two subjects (dynamical meteorology and synoptic meteorology) are of course intimately related and here we make no further attempt to separate them.

More than 2000 years ago meteorology was an integral part of physics. Some would have claimed, “meteorology *is* physics”. Unfortunately, less than a century ago the position of meteorology was less privileged. This is illustrated very well by the following words from the *Feynman Lectures on Physics* (1963) [3].

---

[1] **Zonal average** is the average with respect longitude.

[2] Godske, C.L., T. Bergeron, J. Bjerknes and R.C. Bundgaard, 1957: **Dynamic Meteorology and Weather Forecasting**. American Meteorological Society, Boston and Carnegie Institution, Washington D.C., 800 pp.

[3] R.P. Feynman, R.B. Leighton and M. Sands, 1963: ***The Feynman* Lectures on Physics.** Addison-Wesley Publishing Company. Online edition: http://www.feynmanlectures.caltech.edu. Richard Feynman received the Nobel Prize in Physics in 1965.

"We turn now to *earth sciences*, or *geology*. First, meteorology and the weather. Of course the *intruments* of meteorology are physical instruments, and the development of experimental physics made these instruments possible, as was explained before. However, the theory of meteorology has never been satisfactorily worked out by the physicist. "Well," you say, "there is nothing but air, and we know the equations of motion of air". Yes we do. "So, if we know the condition of air today, why can't we figure out the condition of air tomorrow?" First, we do not *really* know what the condition is today, because air is swirling and twisting everywhere. It turns out to be very sensitive, and even unstable. If you have ever seen water run smoothly over a dam, and turn into a large number of blobs and drops as it falls, you will understand what I mean by unstable. You know the condition of the water before it goes over the spillway; it is perfectly smooth; but the moment it begins to fall, where do the drops begin? What determines how big the lumps are going to be? That is not known, because the water is unstable. Even a smooth moving mass of air, in going over a mountain turns into complex whirlpools and eddies. In many fields we find this situation of *turbulent flow* that we cannot analyze today. Quickly we leave the subject of weather..."[4]

Nowadays things have changed again, despite the fact that turbulent flow is still a seemingly intractable problem. Precisely during the period when Feynman was giving his lectures, research in the fields of dynamical meteorology and climate dynamics was going through phase of explosive development. Now, we would not consider meteorology as a part of geology, but principally as an application of physics and, to a lesser extent, chemistry. W.J. Humphreys of the U.S. Weather Bureau and author of the sizeable volume, "*Physics of Air*", writes in the issue of 30 August 1935 of *Science* (p. 197) that meteorology awoke to alertness with a "galvanic shock" by the invention of the telegraph in the nineteenth century. Humphreys states that,

"Some twenty-two centuries ago meteorology already was accounted an important science; so important indeed that the discriminating Aristotle wrote a sizeable book on it. Then for two millennia the rains came and the winds blew with never an explanation of how or why - curiosity was inhibited by faith and inquiry estopped by authority. Slowly came a drowsy awakening…

Not long ago meteorology was a descriptive subject that required no preparation to study and but a few weeks' time to master, whereas today it ranges nearly the whole field of classical physics with all the mathematics that implies."

Humphreys did not know that the discipline of meteorology would receive another enormous boost within 15 years by the invention of the computer in the 1940's. Results of experiments with the first numerical weather prediction model that could keep up with the actual weather and produce a reasonable weather forecast were published in 1950[5]. Computers made possible a thorough exploration of the solutions of the very complex coupled nonlinear differential equations that govern the thermodynamical behaviour of a fluid such as the atmosphere. These equations were set forth, beginning more than 2000 years ago, by the Greeks philosophers (**conservation of mass**, based on the philosophy that "nothing comes from nothing"), by Isaac Newton in 1687 (definition of, and relation between, force and the product of mass and velocity, called momentum, embodied in **Newton's second law**), by Emile Clapeyron in 1834 (the **equation of state or ideal gas law**) and by Julius von Mayer, James Prescott Joule, Rudolf Clausius and Lord Kelvin (**conservation of energy**, now referred to as the **first law of thermodynamics**) in the mid-nineteenth century. The expression of these laws, in the form of mathematical (differential) equations, was complete at some point in the late nineteenth century, due to the work of, notably, Pierre-Simon Laplace, William Ferrel, Horace Lamb and Wilhelm von Bezold. Cleveland Abbe in 1901 and Vilhelm Bjerknes in 1904 [6] summarised these equations, showing that they

[4] These words of Feynman make clear that the subject of Atmospheric Dynamics is difficult. The laws on which the subject is based have been known since the nineteenth century. However, these laws have provided solutions to atmospheric problems only very slowly, precisely because of this property of "flow-instability".

[5] Charney, J.G., R. Fjørtoft and J. von Neumann, 1950: "Numerical Integration of the Barotropic Vorticity Equation". **Tellus 2** (4): 237.

[6] Abbe, C., 1901: The physical basis of long-range weather prediction. **Mon.Wea.Rev., 29**, 551-561.

form a closed set that can in principle be solved if the state of the atmosphere is known at a certain point in time.

However, Abbe and Bjerknes were too optimistic about the feasibility of this project. The equations set forth by Abbe and Bjerknes form a closed set only if physical processes, such as the emission, absorption and transfer of radiation, phase changes of water and turbulent exchange of energy, momentum and water vapour between the earth's surface and the atmosphere, which drive the atmospheric circulation, and which were not yet fully understood, are neglected. Absorption and emission of radiation by atmospheric constituents, such as water vapour, is at present (2020) still a topic under intense scrutiny!

Lewis Fry Richardson showed, in a remarkable book, entitled "*Weather prediction by numerical process*", published in 1922, how the project, promoted by Abbe and Bjerknes, could be executed numerically. Richardson's ideas, specifically on radiative transfer of energy and turbulent transfer of momentum, heat and humidity, proved very fruitful when they were put to test after 1950. Over the past nearly 70 years the set of general laws of fluid motion, thermodynamics, turbulence and radiation, which were only a skeleton in the days of Abbe and Bjerknes, came alive, thanks to the computer. The (numerical) solutions of these equations are sometimes puzzling and complex[7], while the myriad phenomena that they describe are so rich that one can devote many lifetimes to their investigation. Maybe this is what made Richard Feynman turn away from fluid mechanics (and meteorology) to focus and contribute to the emergence of a totally new understanding of the concepts of mass, energy and momentum, which was initiated by Albert Einstein in 1905 [8]. Here, we stick to the **concepts of mass, energy, momentum and space-time of "classical physics"**, as they were understood by Cleveland Abbe and Vilhelm Bjerknes in 1904.

ii Basic question

The **basic question of these lecture notes** is phrased as follows (following John Dutton[9]).

The basic problem of atmospheric dynamics revolves around the question of why the observed responses are those that are "chosen".

In order to start looking for an answer to this question, we have to define our point of departure. We have to define the **prerequisites**, i.e. the required knowledge to start the study of the dynamics of the atmosphere. You (the reader or student) should be familiar with the basic concepts (momentum, mass en energy) and equations that describe the movement as well as the thermodynamic state (temperature and pressure) of an air-parcel in the atmosphere of a rotating planet, i.e. the concepts and equations that were summarized in 1904 by Bjerknes. You should also be familiar with the basic laws of radiation due to Lambert, Planck, Stefan, Boltzmann and Kirchhoff. This basic knowledge is usually attained in introductory courses in mechanics (equation of motion, Coriolis effect), hydrodynamics (equation of motion, continuity equation), geophysical fluid dynamics (equation of motion, Coriolis effect), classical thermodynamics (equation of state and energy conservation) and radiative transfer. Any person with this basic knowledge of physics should be able to read and understand these notes. Nevertheless, I think some familiarity with meteorology (e.g. from the book by McIlveen (2010) or the book by Wallace and Hobbs (2006), and the first two chapters of Holton (2004)) will be extremely helpful if not essential as a basis to understand the subject matter in these lecture notes (see the list of books in the next section).

Bjerknes, V., 1904a: Das Problem der Wettervorhersage, betrachtet vom Standpunkte der Mechanik und der Physik. **Met. Zeit., 21**, 1-7. The translation of this paper into English by E. Volken and S. Brönniman, with an interpretation by G. Gramelsberger, was published in **Met Zeit (NF),18**, 663-673.

[7] In fact, Bjerknes's dream was "broken" to a certain degree in 1963 when Edward Lorenz (see problem 0.2) questioned its feasibily in an article entitled, "Deterministic nonperiodic flow", published in **Journal of the Atmospheric Sciences**, vol. **20**, 130-141, which showed that the solutions of nonlinear systems with forcing and dissipation can be "chaotic". This means that the solutions depend sensitively on initial conditions.

[8] See the fascinating account by Carlo Rovelli, 2016: **Reality is not what it seems**. Penguin Books, 272 pp.

[9] Dutton, J.A., 1986: **The Ceaseless Wind**. Dover Publications, Inc, Mineola, 616 pp. (see page 6).

iii List of books

The basic theory of fluid dynamics is introduced very well in the book by Batchelor (1970) (see the list below). For the theory of thermodynamics the book by Ambaum (2021) is recommended. Textbooks that treat the basic principals of atmospheric dynamics are those by Holton (1972-2012), Gill (1982), Dutton (1986), James (1994), Salby (2012), Vallis (2017), Martin (2006) and Hoskins and James (2014). The four editions of the book by Holton are probably the most suitable. The books by Martin (2006) and Hoskins and James (2014) are very suitable for learning the basic principles, although these books are more limited in scope compared to Holton's book. The books by Dutton (1986), James (1994), Holton (2004), Vallis (2017) and Hoskins and James (2014) are the most theoretical. However, a substantial part of the theory treated in these notes *cannot* be found in any of the above-mentioned books. Wherever this is true, the references to the pertinent journal articles are given. The books by Lackmann (2011) and Markowski and Richardson (2010) contain many examples of precipiation producing weather systems, such as cyclones and severe convective storms. The book by Markowski and Richardson is beautifully illustrated. Peixoto and Oort (1992) is a "classic" on the physics of the present climate of Earth's atmosphere. More recent advances is our knowledge of the global circulation of the atmosphere are incorporated in the book by David Randall (2015), which next to Andrews (2010) and Salby (2012) is broadest in scope, treating also the radiation balance of the atmosphere, a subject that is usually considered to be part of a separate course on Radiation or on Climate Dynamics.

So, we have the following list of recommended textbooks on Atmospheric Dynamics (the **books listed in red** and underlined are especially **recommended**):

Ambaum, M.H.P., 2021: **Thermal Physics of the Atmosphere second edition**. Elsevier, 259 pp.

Andrews, D.G., 2010: **An Introduction to Atmospheric Physics**. (second edition) Cambridge University Press. Orlando. 237 pp.

Batchelor, G.K., 1970: **An Introduction to Fluid Dynamics**. Cambridge University Press, 615 pp.

Cushman-Roisin, B. and J-M Beckers, 2011: **Introduction to Geophysical Fluid Dynamics: Physical and Numerical Aspects**. Academic Press, New Jersey, 813 pp.

Dutton, J.A., 1986: **The Ceaseless Wind**. Dover Publications, Inc, Mineola, 616 pp.

Gill, A.E., 1982: **Atmosphere-Ocean Dynamics**. Academic Press. 662 pp.

Holton, J.R., 2004: **An Introduction to Dynamic Meteorology**. Fourth edition. Academic Press, 535pp. (The first (1972), second (1979), third (1992) and fifth (2012) edition are also very useful).

Hoskins, B.J., and I.N. James, 2014: **Fluid Dynamics of the Midlatitude Atmosphere.** Wiley Blackwell, 408 pp.

James, I.N., 1994: **Introduction to Circulation Atmospheres**. Cambridge University Press. 422 pp.

Lackmann, G., 2011: **Midlatitude Synoptic Meteorology**. American Meteorological Society. 345 pp.

Markowski, P.M., and Y.P. Richardson, 2010: **Mesoscale Meteorology in Midlatitudes**. John Wiley, 430 pp.

Martin, J.E., 2006: **Mid-latitude Atmospheric Dynamics**. Wiley, 324 pp.

McIlveen, R., 2010: **Fundamentals of Weather and Climate**. Second edition, Oxford University Press, 632 pp.

Peixoto, J.P., and A.H. Oort, 1992: **Physics of Climate**. American Institute of Physics. 520 pp.

Randall, D., 2015: **An Introduction to the Global Circulation of the Atmosphere**. Princeton University Press. 442 pp.

Salby, M.L., 2012: **Physics of the Atmosphere and Climate**. Cambridge University Press, 718 pp.

Vallis, G.K., 2017 **Atmospheric and Oceanic Fluid Dynamics**. Second edition Cambridge University Press. 946 pp.

Wallace, J.M., and P.V. Hobbs, 2006: **Atmospheric Science: An Introductory Survey**. Second edition. Academic Press, 483 pp.

Much interesting material can be found on the web. The classical **Feynman Lectures on Physics**, now available online at http://www.feynmanlectures.caltech.edu, are highly recommended because of the physical insight they convey. Also recommended is Ben Crowell's page, http://lightandmatter.com, from which several interesting books on physics and mathematics (fundamentals of calculus) can be downloaded. A very interesting account of the history of hydrodynamics is found in the book, entitled "**Worlds of Flow**" (Oxford Univesity Press, 2005) by Olivier Darrigol. Finally, an equally interesting account of the more recent history of meteorology is found in the book, entitled "**A Vast Machine**" (MIT Press, 2010) by Paul Edwards.

What justifies the existence and use of these lecture notes when so many good textbooks are available? Well, dear student, the choice of topics as well as their ordering in these lecture notes, which in the beginning followed the book by Holton, has changed slowly over the past 25 years of teaching dynamical meteorology (and other related courses) at the University of Utrecht. This teaching has also determined the direction of my research, which has led to the insight that some often-neglected topics should have a prominent place in a course on the dynamics of the atmosphere. The concept of **potential vorticity** and its use in describing the structure and dynamics of the atmosphere constitutes one of these topics. The interaction between waves and the zonal mean flow is approached here from a "**potential vorticity mixing**" perspective. In most textbooks, except in the recent textbook by Hoskins and James (2014), potential vorticity is treated quite superficially. **Adjustment to hydrostatic balance** and the **interaction between radiation and dynamics** are not considered worthy of much discussion in any textbook on *Atmospheric Dynamics* either, even though a discussion of the latter topic is required to explain the characteristic thermal layering of the atmosphere, i.e the division of the atmosphere into a troposphere and a stratosphere as well as the distribution of potential vorticity.

iv A one-period (7.5 ECTS) masters course on Dynamical Meteorology

These lecture notes are used for a 7.5 ECTS master course on **Dynamical Meteorology** at Utrecht University. At the start of the course it is assumed that the student is more or less familiar with the basics of Geophysical Fluid Dynamics. Students, whose knowledge is deficient in this respect, are strongly advised to do the third year bachelors course on **Geophysical Fluid Dynamics**, which is based on the book by Cushman-Roisin and Beckers (2011). The Masters course on Dynamical Meteorology consists of 2 lectures and 2 tutorials per week. All of these lectures cover **chapter 1** of these notes, which is by far the longest chapter of the existing 12 chapters of the "living document", called "Atmospheric Dynamics"[10]. Chapter 1 treats the full subject of dynamical meteorology at an introductory level. All subsequent chapters are focused on a more in depth, or extensive, treatment of a specific important topic that is introduced in **chapter 1**.

In the **academic year of 2021-2022** the Dynamical Meteorology Course will be given in period 2, starting in week 46 of 2021 and ending in week 5 of 2022 (Christmas holiday is scheduled in weeks 52 and 1; Friday December 24 is also free). The final grade (pass: if ≥5.5/10) is based on **3 projects** and a **final exam**.

The **projects** can be done in groups of up to 3 students. The projects are **project 1** (**problem 1.6** on page 53; 15% of the final grade), **project 3** (**problem 1.14** on page 126; 15% of the final grade) and **project 5** (**problem 1.24** on page 189; 20% of the final grade). These projects belong, respectively, to parts A, E and F of chapter 1 (see pages 13 and 14).

[10] (https://webspace.science.uu.nl/~delde102/AtmosphericDynamics.htm)

The **final exam** (50% of the grade; minimum grade for the final exam: 4/10) consists of questions, representing variations of the following problems treated in the tutorials. Problems 1.1-1.5 and 1.7 of **part A** (p. 52-54), problem 1.8 of (p. 78) **part B**, problems 1 and 2 in **Box 1.8** (p. 88) and problems 1.11-1.12 (p. 89) of **part C**, problems 1.17 and 1.18 (p.127-128) of **part D**, problems 1.20 and 1.21 (p. 153) of **part E**, problem 1.22 (p. 189) of **part F**, problems 1.29, 1.30 (p. 224) and 1.33 (p. 227) of **part G**, problems 1.34, 1.35 and 1.37 (p. 252-253) of **part H** and problem 1.38 (p. 272) of **part I**.

After studying **chapter 1** the student should be familiar with the application of the principals of geophysical fluid dynamics to the atmosphere and with the most important conceptual models of weather phenomena.

The remaining (online) chapters of these lecture notes cover the energy balance and the radiative-convective equilibrium (**chapter 2**), which is part of a second year one-semester Physics Bachelors course that comes under the title **Climate Dynamics**, high frequency waves and hydrostatic adjustment (**chapter 3**), convection and thunderstorms (**chapter 4**), adjustment to geostrophic balance (**chapter 5**), the influence of orography (**chapter 6**), the zonal mean (average around latitude circles) state of the atmosphere, including the jets (**chapter 7**), the tropical Hadley circulation (**chapter 8**), the instability of low frequence planetary waves and cyclogenesis (**chapter 9**) and numerical simulation of the life cycle of middle latitude cyclones (**chapter 10**), the equator to pole transfer of thermal energy (sometimes called "sensible heat") and momentum by planetary waves and large-scale vortices ("eddies") and the influence of this transfer on the zonal mean state (**chapter 11**), the interaction between diabatic processes (e.g. radiation) and adiabatic processes (e.g. transfer of heat and momentum associated with dynamics) in shaping the zonal mean state of the atmosphere (**chapter 12**) and, finally, **chapter 13** is concerned with zonal asymmetries (waves and circulations) in the tropics and with the question how these waves propagate into the extratropics and affect weather there. The material in **chapters 7, 10, 11 and 12** is part of a one-semester graduate course on the transport and mixing in the atmosphere under the title **Boundary Layers, Transport and Mixing**. To assist the student in studying the material, each chapter is concluded with an abstract, listing the key concepts and summarizing the principle conclusions.

Figure 0.1: Launching a radiosonde with Climate Physics master students on 30 November 2012 during an excursion to the Dutch National Weather Service (KNMI) A balloon filled with helium (picture at left) carries an instrument (picture at right) that measures pressure, temperature and humidity up to a height of about 20 to 30 km above sea level. During its ascent to a height of 20 to 30 km above sea level, the balloon is tracked by radar. The wind-vector as a function of pressure is determined from the position of the balloon.

v Weather maps, observational data and reanalysis data

There exists an immense amount of information about the weather, including observational data, processed data in the form of weather maps, forecasts made by modelling centres around the world and "**reanalysis**" of past observations. Since the beginning of the twentieth century the three-dimensional structure of the atmosphere has been monitored by **surface observations**, by **radiosondes** (**figure 0.1**), which were rare before 1950, and from the 1970's onwards by **satellite derived observations**. These observations are available in raw form or in plotted form on many websites. The radiosonde network consists of about 1500 stations, which are distributed rather unevenly over the world (**figure 0.2**). Many of these stations are hardly in operation. During 2003, for instance, less than half of these stations were active on 80% of the days (**figure 0.3**). About 400 stations were not in operation at all in 2003.

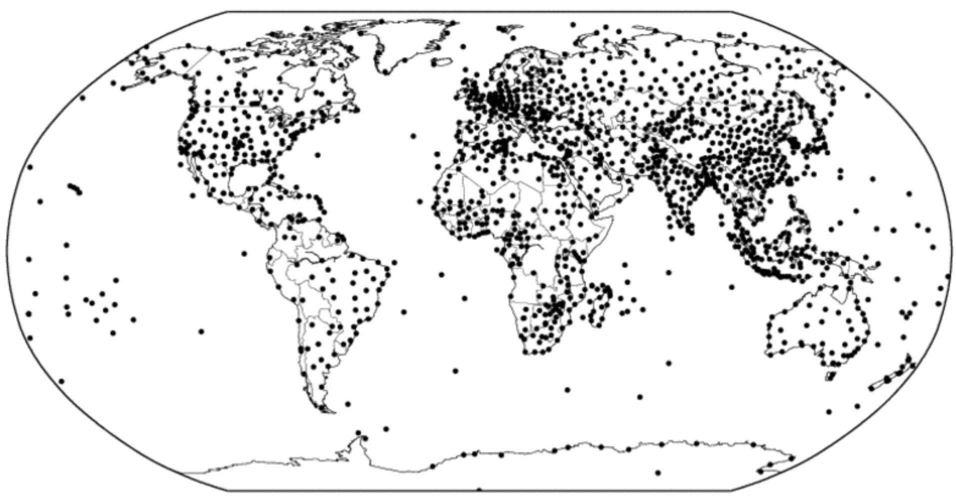


Figure 0.2: Locations of all stations in IGRA (Integrated Global Radiosonde Archive) (http://www.ncdc.noaa.gov/oa/climate/igra/).

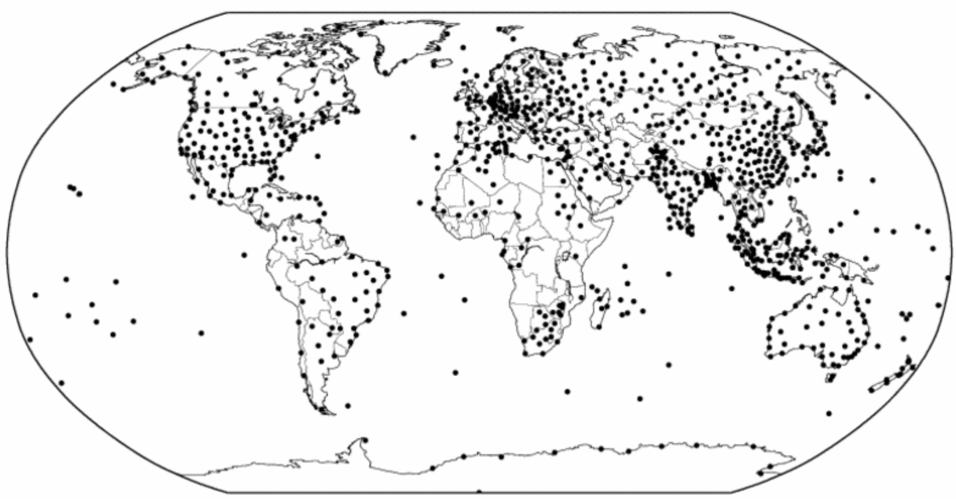


Figure 0.3: Locations of stations in IGRA, which were active during 2003.

The reanalyses of raw observations consists of using a numerical weather prediction model together with a data assimilation scheme in an analysis/forecast cycle with the aim to produce an internally consistent 4-dimensional analysis of the state of the atmosphere over many decades. The introductory page (before 2013) on the website of the **European Centre for Medium Range Weather Forecasts** (**ECMWF**) reanalysis project starts with the following explanation [11]:

[11] see http://www.ecmwf.int/en/research/climate-reanalysis

"Reanalyses of multi-decadal series of past observations have become an important and widely utilized resource for the study of atmospheric and oceanic processes and predictability. Since reanalyses are produced using fixed, modern versions of the data assimilation systems developed for numerical weather prediction, they are more suitable than operational analyses for use in studies of long-term variability in climate. Reanalysis products are used increasingly in many fields that require an observational record of the state of either the atmosphere or its underlying land and ocean surfaces. Estimation of renewable energy resources, calculation of microwave telecommunication signal losses and study of bird migration are just three examples. The first reanalysis at ECMWF was carried out in the early 1980s for the First GARP Global Experiment (FGGE) year 1979, when ECMWF operations began. Two major ECMWF reanalyses have exploited the substantial advances made since then in the forecasting system and technical infrastructure. The first project, ERA-15 (1979-1993), was completed in 1995 and the second extended reanalysis project, ERA-40 (1957-2002), in 2002. Products of ERA-15 and ERA-40 have been used extensively by the member states and the wider user community[12]. They are also increasingly important to many core activities at ECMWF, particularly for validating long-term model simulations, for helping develop a seasonal forecasting capability and for establishing the climate of EPS (Ensemble Prediction System) forecasts needed for construction of forecaster-aids such as the Extreme Forecast Index. ECMWF is currently producing ERA-Interim, a global reanalysis of the data-rich period since 1989. The ERA-Interim data assimilation system uses a 2006 release of the Integrated Forecasting System (IFS Cy31r2), which contains many improvements both in the forecasting model and analysis methodology relative to ERA-40. The ERA-Interim reanalysis caught up with operations in March 2009, and is now being continued in near-real time to support climate monitoring".

Indeed, the arrival of reanalysis products in the 1990's has ushered a revolution in Dynamical Meteorology. Unfortunately, this is a slow revolution. There is a wealth of information in the reanalysis products, which is still lying dormant. This is because, since the 1990's, fundamental research in Dynamical Meteorology has suffered from the excessive attention of the research community to the problem of global warming.

Meanwhile, reanalysis projects are being carried out by ECMWF ("ERA-40" and "ERA-Interim") (ERA-Interim: https://www.ecmwf.int/en/forecasts/datasets/reanalysis-datasets/era-interim/), by the National Center for Atmospheric Research (NCAR) (see the NCAR Climate Data Guide (https://climatedataguide.ucar.edu/climate-data/atmospheric-reanalysis-overview-comparison-tables/) and NCEP/NCAR), by the Japan Meteorological Agency ("JRA-55") and **NASA** ("MERRA"). The Japanese reanalysis project and the ERA-40 reanalysis have both yielded a fascinating atlas, which can be viewed at JRA-55 Atlas (https://jra.kishou.go.jp/JRA-55/atlas/en/index.html) and at ERA-40 Atlas. The most recent reanalysis projects are the 20th century reanalysis of the NOAA Physical Science Division (PSD) (NOAA-CIRES Twentieth Centrury Reanalysis (https://psl.noaa.gov/data/gridded/data.20thC_ReanV2c.html)the 20th century reanalysis of ECMWF (ERA-20C) at and the high resolution ERA-5 reanalysis at https://cds.climate.copernicus.eu/cdsapp#!/home.

Reanalysis data is usually in **netCDF-format**. The dataviewer, **PANOPLY**, can be used to view the data. This program can be downloaded freely from http://www.giss.nasa.gov/tools/panoply/. Information about the netCDF-format can be found on http://www.unidata.ucar.edu/software/netcdf/. You can also read and process netCDF-data with the programming language, **PYTHON** (https://www.python.org), which is rapidly becoming a very popular language to analyse "big data" sets [13]. The book, written by Johnny Lin, entitled "**A Hands-On Introduction to Using Python in the Atmospheric and Oceanic Sciences**" is recommended (http://www.johnny-lin.com/pyintro/). Archived weather maps can be viewed on http://www.wetter3.de/Archiv/. Radiosonde observations since the 1970's can be retrieved from http://weather.uwyo.edu/. A longer list of websites providing weather information can be found at http://www.staff.science.uu.nl/~delde102/WeatherDiscussions.htm.

---

[12] The state of the art *ERA- 5* reanalysis, which runs from 1 January 1979 to *present*, and its predecessor, *ERA-Interim* reanalysis, which runs from 1 January 1979 to August 2019, are recommended.

[13] http://www.nature.com/news/programming-pick-up-python-1.16833#/related-links .

The main text of these lecture notes is interspersed with **problems** and "**Boxes**". The **problems** are not limited to manipulating and deriving equations, but are also frequently very practical, intending to aid the student in learning and practicing skills in physical interpretation, computer programming, data-collecting and statistical analysis. "**Boxes**" summarise prerequisite knowledge, give stand-alone derivations, which are referred to frequently in different chapters, or give information, which is relevant but a bit outside the scope of Atmospheric Dynamics.

PROBLEM 0.1 Predictability in a non-linear mathematical model
The atmosphere, the ocean and climate are governed by a system of **non-linear** differential equations. As an example of such a system (presented first by Edward Lorenz in 1984), we study the following very simple model.

$$X(t+1) = aX(t) - X(t)^2 \ .$$

This is a non-linear recurrence relation, which is quite similar in structure to a "finite difference" numerical approximation of a differential equation that governs the behaviour of a fluid, such as the atmosphere or the ocean. Think of t as representing time, X as representing the global average surface temperature and a as representing the CO_2 concentration. What is the equilibrium value of X? Write a program or script (e.g. in Python) which calculates and plots the evolution of X in time (t) for the following 4 values of a: a=1, a=2, a=3.25 or a=3.75 (take $X(0)=0.5$) . Discuss the results. Define the "weather" and the "climate" of this model. For which values of a, is the model-"weather" "predictable"? For which values of a, is the model-"climate" "predictable"?

PROBLEM 0.2 Making a weather forecast
Every week (on Wednesday) during the course we shall make a weather forecast for the following Sunday. The weather parameters of interest are daily maximum/minimum temperature, sunshine duration, as a percentage of the maximum possible, and precipitation. In order to simplify the evaluation of this forecast, we make the forecast categorical. A categorical forecast consists of a flat statement that can be either true ("hit") or false ("miss"). For example a preciptation forecast, which states that there will be no rain during one day, can either be true or false. Likewise, a temperature forecast that states that the temperature will be normal can be true or false, if we define "normal" and "abnormal". Here, we define a "normal temperature" as lying in the range $T_{mean}-\Delta T<T<T_{mean}+\Delta T$, where T_{mean} is the **mean** temperature on that particular day over the past, say, 30 years, while ΔT is equal to some fraction of the **standard deviation**, σ, of these temperature recordings. An "abnormal temperature" would then correspond to $T<T_{mean}-\Delta T$ or $T>T_{mean}+\Delta T$. By introducing the standard deviation, we assume that the probability density distribution of temperature recordings over the past 30 years is symmetric about the mean and is Gaussian. This is usually not a good approximation to the true probability distribution. It is better to deduce the range of "normal temperatures" from the **cumulative distribution function**, $P(T\leq x)$, which describes the probability that the temperature, T, is lower than, or equal to x. We divide the distribution of observed values of T into three "bins" (below normal, normal and above normal), where "normal" represents the range of values $T_1<T<T_2$ such that $P(T\leq T_2)-P(T\leq T_1)=1/3$, while $P(T< T_1)=1/3$, so that, obviously $P(T> T_2)=1/3$.